Number line and simple fractions

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Introduction

Development of insight in rational numbers
refers to changes in the correspondences between meanings and signs. In the educational field the choice of representations is mostly centered on enactive or iconic representation. Number line as a highly abstract (iconic-symbolic) « Mitteilungszeichen » (sign of message, Nietzsche; sign of relation, Peirce) is less explored. Number line seams to be an indicator of the higher levels of the « abstraction réfléchissante » of fractions, a kind of red thread of understanding numbers and mathematical education.

Research aims and hypotheses

Since 1980 it has been well documented that students even at the age of 12 find difficulty in using number lines to work with fractions (Watanabe, 2002; Padberg, 2002). Watanabe concludes «…that number lines do not help students develop a sense of fractions as numbers but that number-line representations make sense only to those students who already understand fractions as numbers » (p. 462).

Sinclair et al. (1988) reported how preschool children create and read notations of natural numbers. In Brizuela's (2006) interesting study kindergarten- and first grade children had to explain their notations for fractions and they had to show the different numbers on the number line. Brizuela found three groups of meanings of fractions: « half is a little bit »; different understandings across different contexts (partitioning cookies or pizzas); similar understanding across different contexts.
Young children generates meanings for fractional numbers, number lines and contexts. There must be bridges of arguments in the « abstraction réfléchissante » between the natural and the rational numbers.

Our study explored the development of correspondences between simple fractions and the number line. What do children know about number line and what kind of conceptual arguments will be produced for ordering a mixed number (1 ½)?
We also explored if the development of insight correlates with grades of schooling, types of classes (including special education) and sex.

Differing from Brizuela we excluded contextual manipulatives and concentrated on the number line. Differing from Watanabe we postulate that every correspondence with the number line makes sense, not only the right understanding of some numbers. Differing from Moss & Case (1999) number line is an open tool rather than an object of ordered training.

EXPERIMENT

Participants

90 children ranging in age from 4;11 years to 15;5 years (*M*= 9;5) were clinically interviewed. Master students of the University of Applied Sciences of Special Needs Education were introduced in this method of clinical interviewing.

Procedure

The pretest. Every child passed a pretest drawing and explaining a number line: "Draw a number line and tell me what it is."

When children had few or no ideas about the number line, the interviewer could offer 3 differently detailed information's.

The experimental Task. After the pretest, all participants completed one experimental tasks: the fraction task:

"Look, here on the number line is 1, there is 2. I ask you now: which of these cards belongs between 1 and 2?"

(move a finger between 1 and 2 on the number line)

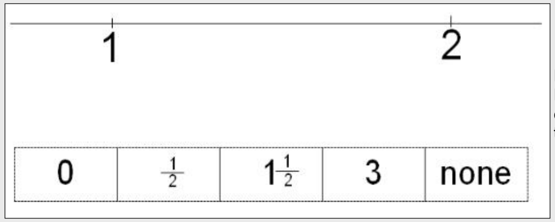

Fig. 1
Number line and number cards in the experimental task

Which is it... ? (point with the finger at the cards 0, ½, 1 ½ , 3) - or does nothing go between? (point at the "none-card")
Or: What belongs between the 1 and the 2? (Child moves a card)

Explain (tell) me: why did you take that card?

RESULTS

36 % of all subjects knew spontaneously what the number line is.

In the randomly selected sample of 44 subjects (see Fig. 2) we found a large correlation between the performance in the pretest (concept of number line) and the knowledge of simple fractions (*Kendalls τbis (tau - biserial)* = -.55, *p* < .05).

The big majority of the children in the "minus" - number line–group (70% of n=44) was not able to understand and to place 1 ½ on the number line.

The first offer of information (drawing just a line) effected that 13 children could order the mixed fraction correctly. The second offer (drawing a line and number 1 to 3) helped 5 children. The last offer (drawing a line and number 1 to 10) was helpful for 2 children. Differing from Watanabe (2002) talking about number line helps to understand a mixed fraction (Moss & Case (1999) .

The age of the children and the grades correlated also with the insight in simple fractions. No correlations has been observed between the types of schooling, sex and the understanding of simple fractions.

Levels of Insight or Correspondence

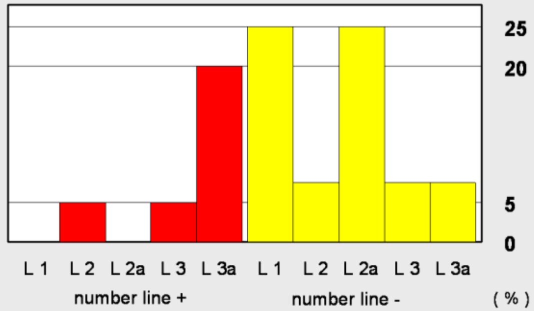

Fig. 2 Number line and levels of understanding 1 ½ (n=44)

5 levels of hypothetic constructs of correspondences of perceptions and logico-arithmetical reasoning were found:
Level 1 represents answers about perceptions of the material, there is no insight in the number line and the given set of numbers.
Level 2 is defined by the counting- and comparison-scheme of natural numbers. The symbols of the fractions are not integrated. There are also arguments about addition of natural numbers.
Level 2a integrates experiences with scales (meter) or with the partitioning of cookies in combination with the symbols of fraction. The cardinality of the fractions is not developed.
Level 3 integrates the correct seriation (counting and cardinality) of the natural and the rational numbers on the number line. 1 ½ is explained as the half between 1 and 2.
Level 3a contains the perfect understanding of the presented fractions in combination with logico-arithmetical operations (part-whole-relation, addition, multiplication, or division). 1 ½ can be correctly explained as a decimal.

Discussion

Our results support Parrat-Dayan's (1980) and Brizuela's view that understanding of fraction is a gradual process. In the setting of a clinical (flexible) interview children constructed logico-mathematical correspondences.
The differences or the correctness of understanding conventional notations could be classified in different levels. The levels represent a growing complexity of insight in natural and rational numbers and operations.
The use of the number line provokes operations (handling, reasoning) and insight in the correspondence between mental and iconic-symbolic representation.
Children constructed their own aspects of fractions on the topic of the number line. They used natural numbers to explain the mixed fraction (counting). They interpreted parts of the conventional notations. They also used seriation, scales, arithmetic operations, part-whole-thinking (Saxe et al., 2005) and decimals.

Developing and exploring this clinical interview offered a psychological view on the development of constructing correspondences (Piaget et al., 1990).
Pragmatic consequences for the research and the education of mathematics should be:
-Use number line as an open tool not as a manipulative
-Enhance research of logico-arithmetical reasoning about fractions in the classrooms
-Root mathematical education (dialogue, cooperation, games) in children's constructions of correspondences rather than rooting in manipulatives.

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